ME 323: FLUID MECHANICS-II

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Lecture-05

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Choking Phenomena

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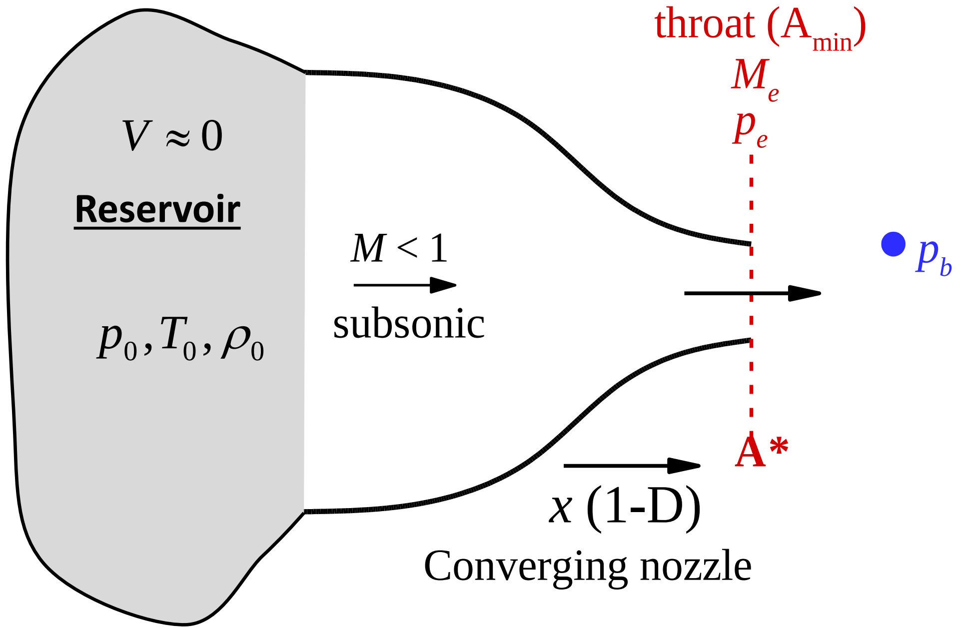
Choking Phenomena

Consider, 1-D isentropic flow through a converging nozzle. A gas is expanded from a reservoir with thermodynamic conditions of p_0 and T_0. The **back pressure** where the gas is **exhausted** is at static pressure of p_b.

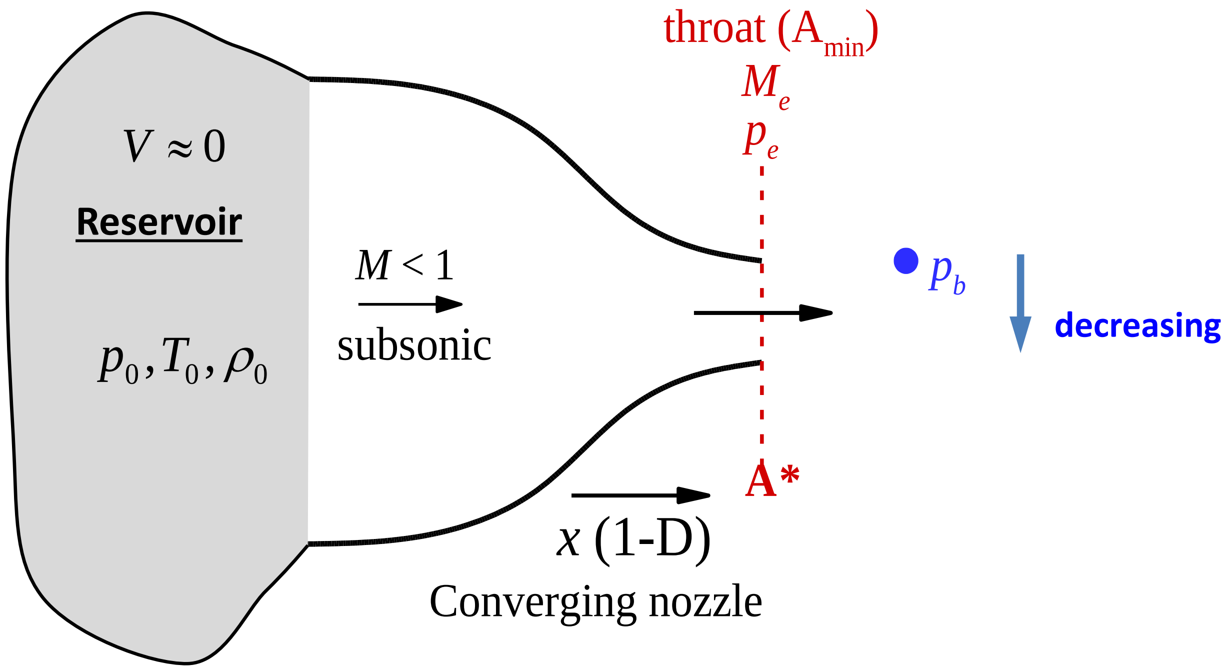
The nozzle exit conditions are M_e and p_e (static).

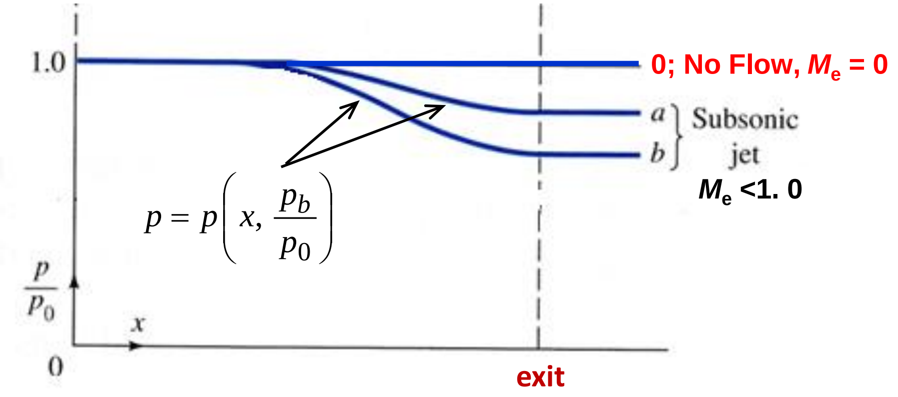


- when $p_b = p_0$; there will be no flow inside the nozzle.
- when $p_b < p_0$ (or $p_0 > p_b$); a flow will be induced from the reservoir and a jet with velocity M_e will be exhausted.
- The magnitude of jet Mach number (M_e) will be increased with the increase of pressure difference (p_0-p_b).

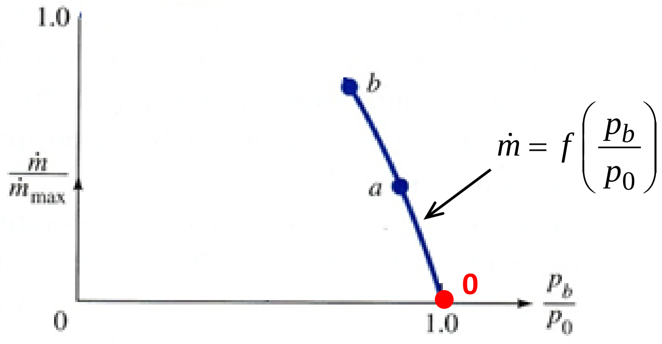
- The reservoir senses the pressure at the exhaust (p_b) and induce the corresponding mass from the reservoir.
- So, mass flow rate will increase with increment of (p_0-p_b).

Choking Phenomena cont...





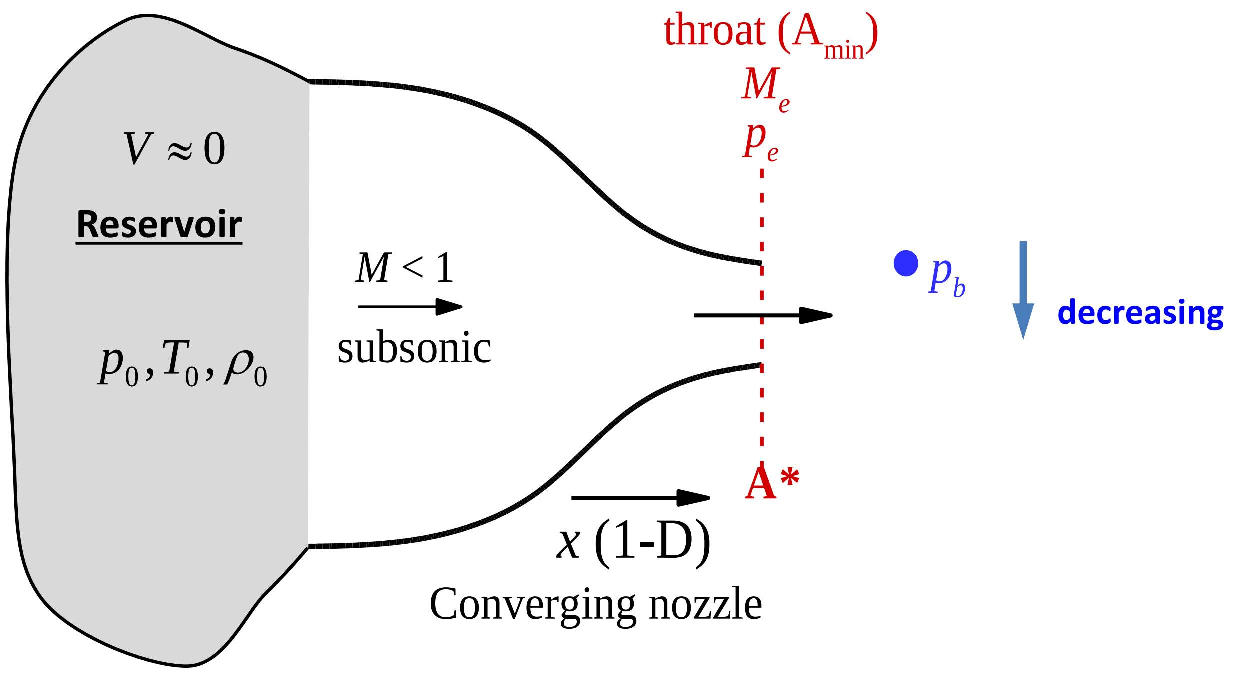
Pressure distribution at difference back pressures (p_b)

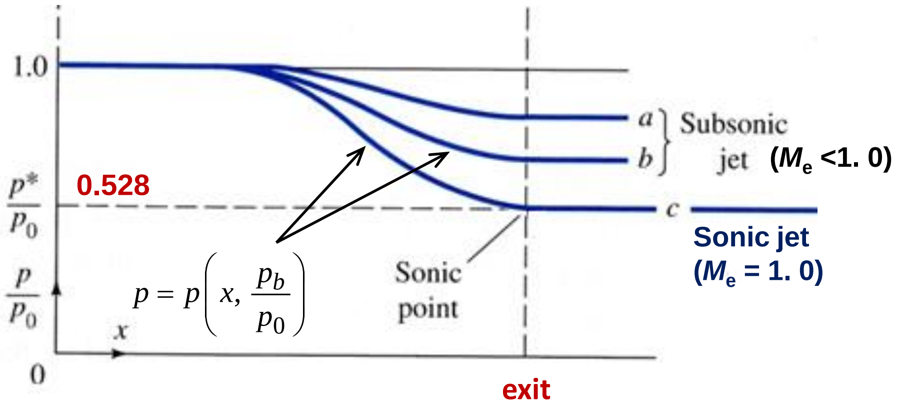


Mass flow rate vs back pressure

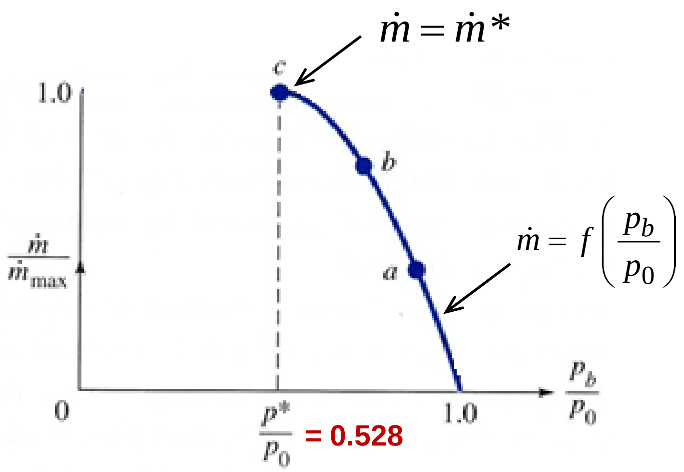
Choking Phenomena cont...

- However, there is a **limit** upto which the mass flow rate will induce.
- The mass flow rate will be the maximum (critical) when the Mach number at the nozzle exit is **sonic** i.e. $M_e = 1.0$.





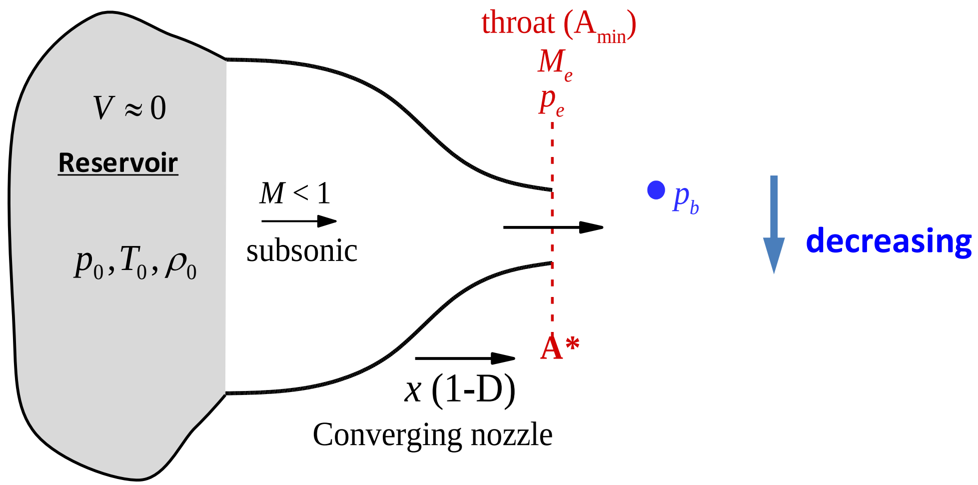
Pressure distribution at difference back pressures (p_b)

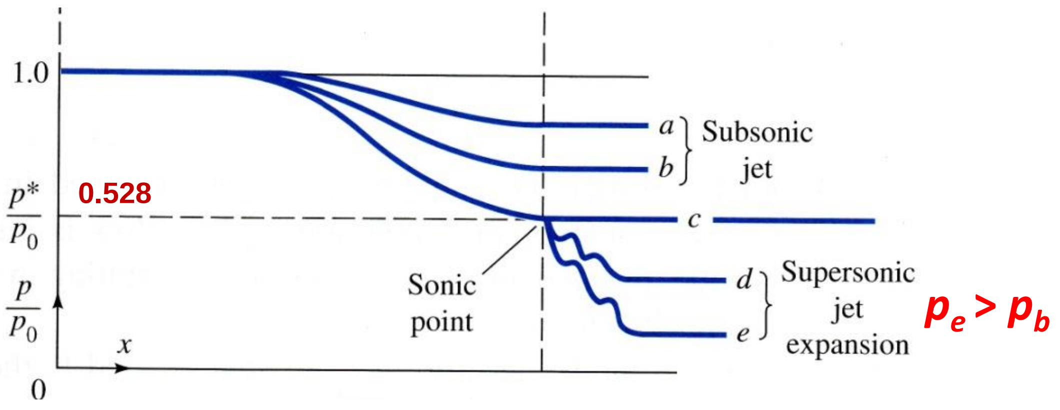


Mass flow rate vs back pressure

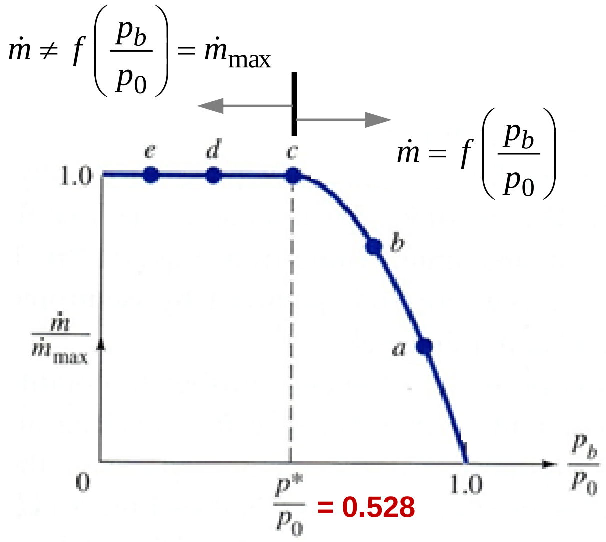
Choking Phenomena cont...

- Further decrease in back pressure (p_b) will not result an increase in flow rate.
- The maximum mass flow rate, m_{max} will be occurred when $M_e = 1.0$.
- This is called the **"choking"** phenomena.
- upto choking condition, $p_e = p_b$





Pressure distribution at difference back pressures



Mass flow rate vs back pressure

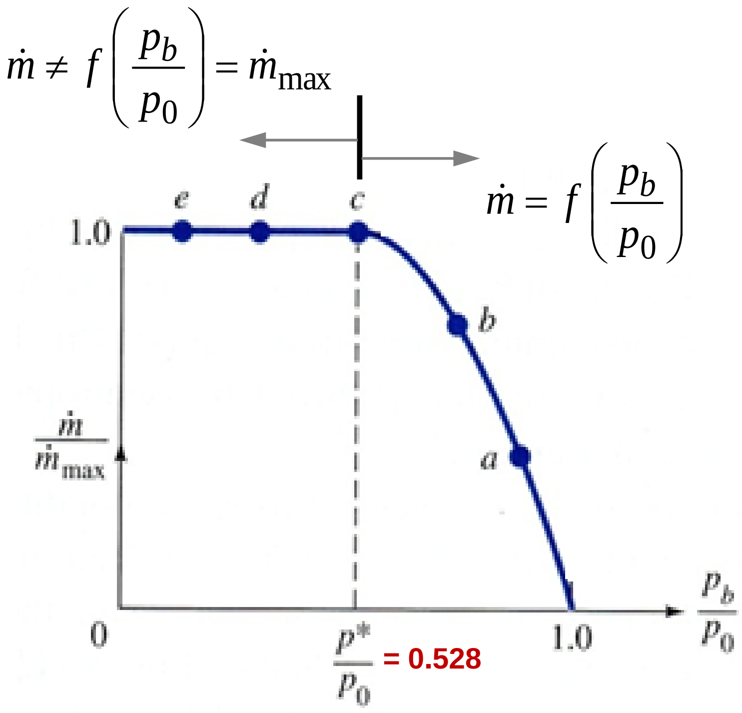
Choking Phenomena cont...

Once the flow reaches the sonic (critical) condition at throat (exit), it becomes “**deaf**” to downstream condition.

Any change (i.e. a reduction) in the applied back pressure (p_b) propagates in the fluid at the speed of sound in all directions, so it gets “**washed**” downstream by the fluid which is moving at the speed of sound at nozzle exit.

Jet cannot send any signal upstream to the reservoir (*or to influence the choked flow conditions*); consequently, **no more fluid is induced from the reservoir**.

For a fixed size of the throat, the maximum mass flow rate through the nozzle is determined by the choked condition.



Choking Phenomena cont...

The **maximum possible mass flow rate is**;

$$\dot m_{\max} = \rho^* A^* V^*$$

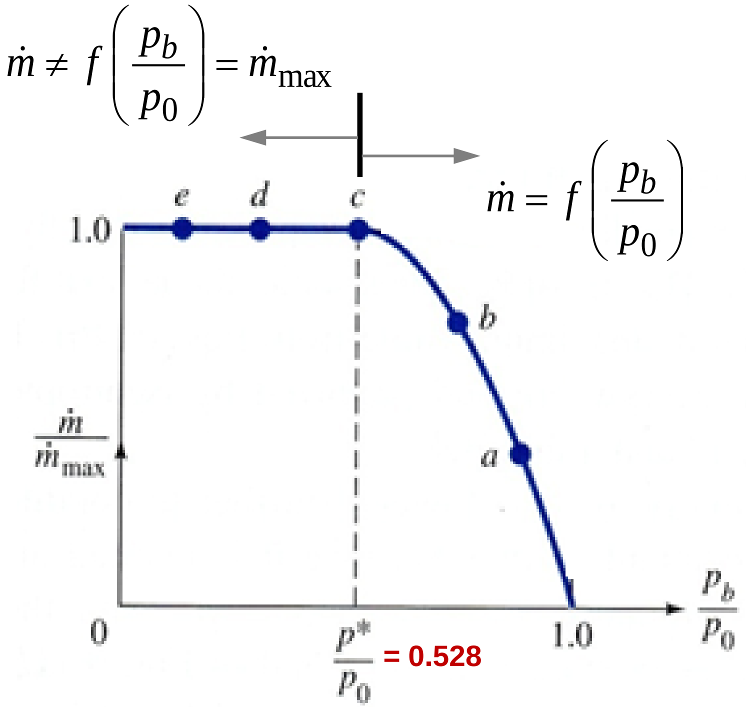
$$\Rightarrow \dot m_{\max} = \rho_0 \left(\frac{2}{k+1}\right)^{1/(k-1)} A^* \left(\frac{2k}{k+1} RT_0\right)^{1/2}$$

$$\Rightarrow \dot m_{\max} = \frac{p_0}{RT_0} \left(\frac{2}{k+1}\right)^{1/(k-1)} A^* \left(\frac{2k}{k+1} RT_0\right)^{1/2}$$

$$\Rightarrow \dot m_{\max} = k^{1/2} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} A^* \frac{p_0}{\sqrt{RT_0}}$$

$$\because V^* = M^* a^* = (1.0)\sqrt{kRT^*}$$

$$\Rightarrow V^* = \sqrt{\frac{2k}{k+1} RT_0} \quad ; \frac{T^*}{T_0} = \frac{2}{k+1}$$



For air; k = 1.4 and R = 287 J/kgK:

$$\boxed{\dot m_{\max} \approx 0.04 \frac{p_0 A^*}{\sqrt{T_0}} \qquad (\text{kg/s})}$$

For isentropic flow through a duct;

the maximum mass flow possible is

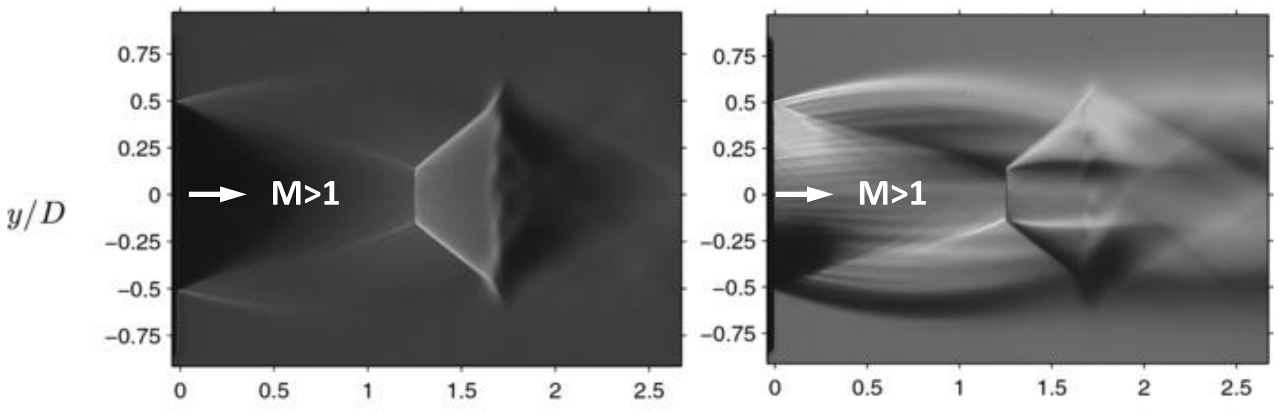
- **proportional to the throat area, A***
- **proportional to stagnation pressure, p_0** and
- **inversely proportional to the square root of the stagnation temperature, T_0.**

Choking Phenomena cont...

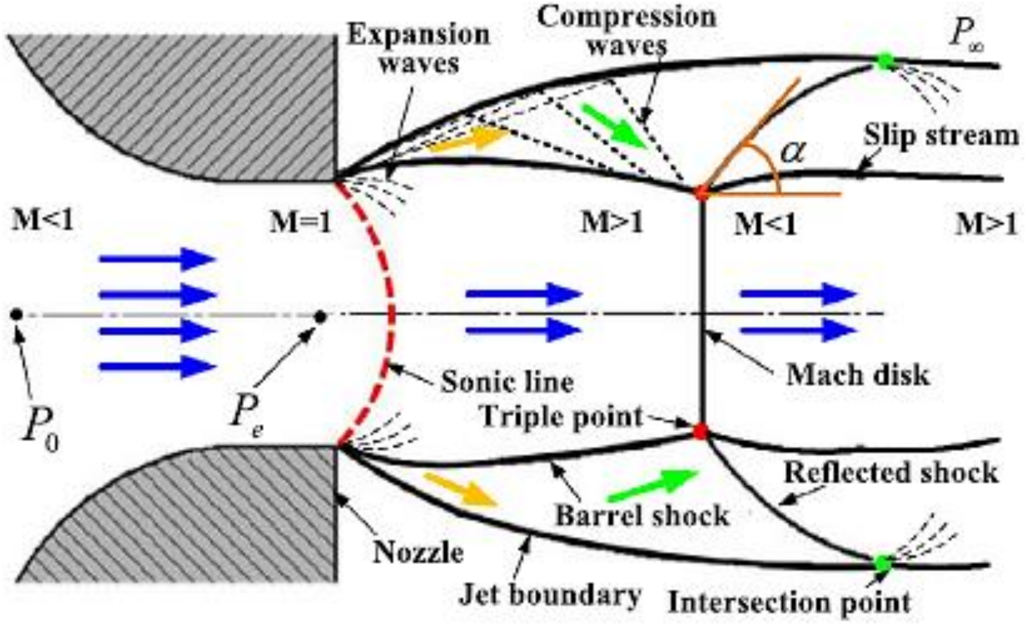
What will happen when

$$p_e > p_b \quad \left(\text{or } p_b < p_e\right)$$

- **Mass flow rate will not increase but will remain fixed as maximum as at choked condition.**
- The exit jet will expand supersonically (M>1) outside the nozzle.
- The jet structure will be very complex with appearance of various waves (expansion and shocks).
- The condition is known as **underexpansion**.



Underexpanded jet

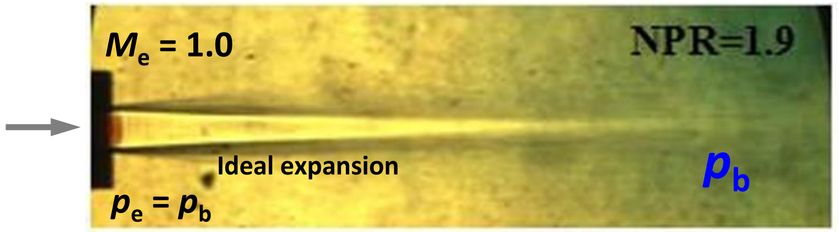


Source: : Phys. Fluids 26, 096101 (2014)
Int J Heat Fluid Flow (2013) 44: 140-154

Underexpanded jet

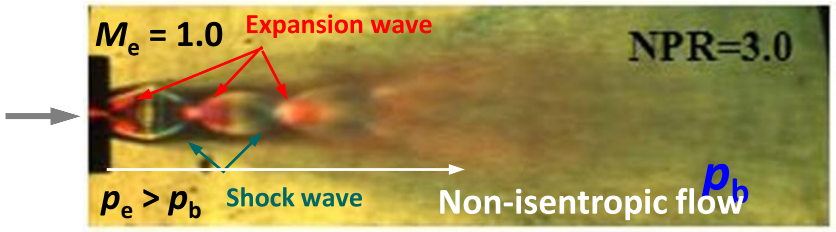
Choking Phenomena cont...

Experimental Visualization through **Schlieren Optical Imaging**

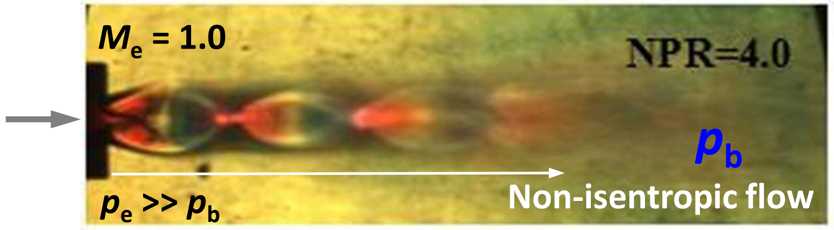


$$\text{NPR} = \text{Nozzle PressureRation} = \frac{p_0}{p_b}$$

$$\rightarrow \frac{p_b}{p_0} = \frac{1}{\text{NPR}} = 0.526 \approx 0.528$$



$$\rightarrow \frac{p_b}{p_0} = \frac{1}{\text{NPR}} = 0.333 < 0.528$$



$$\rightarrow \frac{p_b}{p_0} = \frac{1}{\text{NPR}} = 0.250 < 0.528$$

$p_e > p_b$

Underexpanded

$$\boxed{T = \dot{m}V_e + (p_e - p_b)A_e}$$

ideal **underexpansion**

Source: Behrouzi and McGuirk; AIAA J. (2015)
DOI: 10.2514/1.J053376

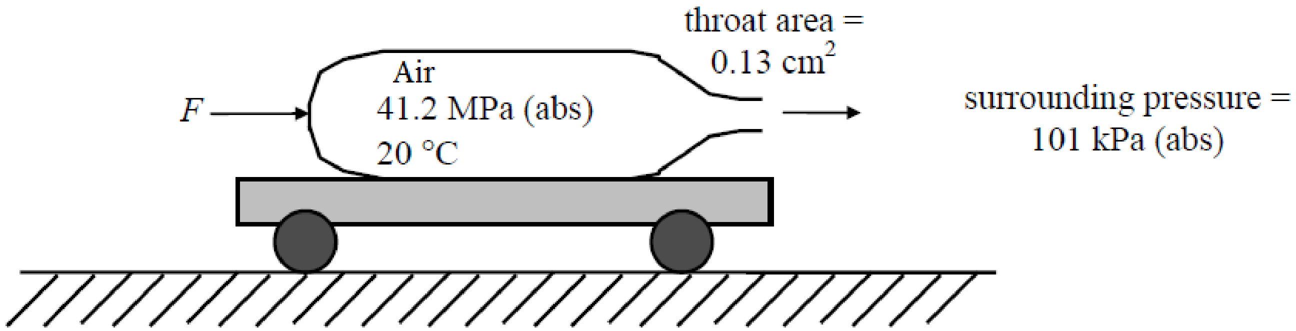
Choking Phenomena

cont...

Problem:

An Air cartridge is used to propel a small rocket cart. Compressed Air, stored at a pressure of 41.2 MPa (abs) and a temperature of 20°C, is expanded through a smoothly contoured converging nozzle with a throat area of 0.13 cm^2. Assume that the cartridge is well insulated and that the pressure surrounding the cartridge is 101 kPa (abs). For the given conditions,

a. Calculate the pressure at the nozzle throat.
b. Evaluate the mass flow rate of air through the nozzle.
c. Determine the force, *F*, required to hold the cart stationary. (Assume frictionless roller)
d. For what range of cartridge pressures will the flow through the nozzle be choked?



To be solved

(a) pe = 21.77 MPa
(b) 1.25 kg/s
(c) 673 N
(d) 191.2 kPa < pc < 41.2 MPa

Choking Phenomena cont…

Problem:

A convergent nozzle has an exit area of 6.25 cm^2. Air enters the nozzle with a stagnation pressure of 0.7 MPa and a stagnation temperature of 95°C. Determine the mass flow rate and jet exit velocity if it is discharging to an environment at (i) 0.6 MPa, (ii) 0.1 MPa, and (iii) 0 Pa.

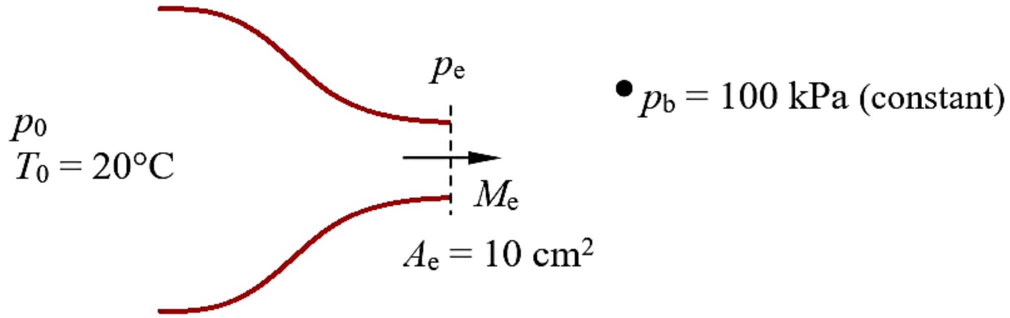
Ans: (i) 0.66 kg/s ($p_b/p_0 = 0.857 > 0.528$ (not choked), $M_e = 0.47$, $p_e = 0.6\text{MPa}$, $V_e = 176.8$ m/s, $T_e = 352.4$ K, $\rho_e = 5.93$ kg/m^3)

(ii) 0.92 kg/s ($p_b/p_0 = 0.143 < 0.528$ (choked), $M_e = 1.0$, $p_e = 0.37\text{MPa}$, $V_e = 351$ m/s, $T_e = 306.5$ K, $\rho_e = 4.21$ kg/m^3)

(iii)

Problem

Air is being discharged to atmosphere (p_b = 100 kPa) through a converging nozzle as shown in figure. The air is being feed from a large reservoir in which the pressure is continuously increased from 200 kPa to 600 kPa. During this operation, the reservoir temperature is maintained constant at 20°C.



(a) Complete the table:

p_0 (kPa)	200	300	400	500	600
p_e (kPa)					
M_e					
$\dot{m}$ (kg/s)					

(b) Plot $\dot{m}$ vs. p_0

(c) Is the nozzle choked or not? Justify your comment.